

Definition: If a variable y depends on a variable x in such a way that each value of x determines at most one value of y , then we say that y is a **function** of x .

Notes:

- For every input (x), there is at most one output (y).
- Not every possible input need be an actual input value. For instance, if we take the “principal” square root function (remember that a square root can give two values, one positive and one negative – the principal root is the positive root) we know that $x = -5$ is not in the domain of the function... so $x = -5$ is not an input value for the function.

Ways to Represent a Function:

1. A table or spreadsheet.
2. Algebraically with an equation.
3. Geometrically with a graph.
4. Verbally.

In each of the cases above, how do we tell if something is a function? (For graphs, this is where we need the “vertical line test.”)

The Absolute Value Function

The absolute value function of a real number is given by

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}.$$

Notice that this is a “piecewise defined function.” In class, we will discuss arithmetic properties of the absolute value function:

$$|-a| \quad |ab| \quad |a/b| \quad |a+b|$$

where we write each in terms of the absolute values of a and b individually. We will also discuss the function

$$f(x) = \sqrt{x^2} = |x|.$$

Some topics that will be discussed in class together are:

1. Piecewise defined functions that are more complicated.
2. Domain and range. (The book mentions “natural domain.”)

3. Open and closed intervals/notation.

Examples:

1. Find the domain and range of $f(x) = \pi + \sqrt{x - 2}$.
2. Find the domain and range of $g(x) = (x + 1)/(x - 1)$.
3. This is example 9 from the text: An open-top box is to be made from a 16-inch by 30-inch piece of cardboard by cutting out squares of equal size from the four corners and folding.
 - (a) Let V be the volume of the box that results when the squares have sides of length x . Find a formula for V as a function of x .
 - (b) Find the domain of V .
 - (c) What does the graph of V look like? More importantly, what does the graph tell you about the volume?