Rational Expressions and Long Division

While Section 1.4 doesn't mention rational expressions, we're going to be needing them quite a bit. Today in class, I discussed rational expressions (as well as the topics in Section 1.4, which you should read) and long division. I am including here some more examples of long division of polynomials.

1. Use long division to express

$$f(x) = \frac{x^3 + x^2 - 1}{x - 1}$$

as a polynomial plus a simpler rational expression.

Solution: We start by setting up the long division. Remember to write the polynomials in standard form so that the terms are listed in decreasing order based on their degrees. Since the polynomial $x^3 + x^2 - 1$ has no degree–1 term, we'll leave a space to help us keep things organized. We start with:

$$\begin{array}{r|rrr} x-1 & x^3 & +x^2 & -1 \\ \hline \end{array}$$

From here, we're wondering what we have to multiply the first term of $x - 1$ by in order to get the first term of $x^3 + x^2 - 1$. That is, what do we have to multiply x by in order to get x^3? The answer is clearly x^2, and so we have

$$\begin{array}{r|rrr} & & x^2 & \\ \hline x-1 & x^3 & +x^2 & -1 \end{array}$$

Multiplying $x - 1$ by x^2 and then subtracting the result from $x^3 + x^2 - 1$ (remember to subtract the entire polynomial!) we get

$$\begin{array}{r|rrr} & & x^2 & \\ \hline x-1 & x^3 & +x^2 & -1 \\ & -x^3 & +x^2 & \\ \hline & & 2x^2 & \end{array}$$

Now continuing we have

$$\begin{array}{r|rrrr} & & x^2 & +2x & +2 \\ \hline x-1 & x^3 & +x^2 & & -1 \\ & -x^3 & +x^2 & & \\ \hline & & 2x^2 & & \\ & & -2x^2 & +2x & \\ \hline & & & 2x & -1 \\ & & & -2x & +2 \\ \hline & & & & 1 \end{array}$$

And so we have

$$\frac{x^3 + x^2 - 1}{x - 1} == \frac{(x^2 + 2x + 2)(x - 1) + 1}{x - 1} = \frac{(x^2 + 2x + 2)(x - 1)}{x - 1} + \frac{1}{x - 1} = x^2 + 2x + 2 + \frac{1}{x - 1}$$