Use long division to rewrite the following rational expressions as polynomials plus simpler rational expressions:

1. Here is a basic one:

$$\frac{x^2 - 3x - 10}{x + 2}$$

Solution:

$$\begin{array}{r|l} & x \quad -5 \\ \hline x+2 & x^2 - 3x - 10 \\ & -x^2 - 2x \\ \hline & \qquad -5x - 10 \\ & \qquad 5x + 10 \\ \hline & \qquad\qquad 0 \end{array}$$

The long division above shows that $x + 2$ is a factor of $x^2 - 3x - 10$. Thus, we have

$$\frac{x^2 - 3x - 10}{x + 2} = \frac{(x + 2)(x - 5)}{x + 2} = x - 5.$$

You could, of course, have simply used factoring to reduce the quadratic and you'd get the same exact answer.

2. This one is slightly longer:

$$\frac{x^4 + x + 1}{x + 1}$$

Solution:

$$\begin{array}{r|l} & \phantom{-x^4 - {}} x^3 - x^2 + x \\ \hline x+1 & x^4 \qquad\qquad\quad + x + 1 \\ & -x^4 - x^3 \\ \hline & \qquad -x^3 \\ & \qquad x^3 + x^2 \\ \hline & \qquad\qquad x^2 + x \\ & \qquad\qquad -x^2 - x \\ \hline \end{array}$$

The remainder here is just 1, and so we have

$$\frac{x^4 + x + 1}{x + 1} = \frac{(x + 1)(x^3 - x^2 + x) + 1}{x + 1} = x^3 - x^2 + x + \frac{1}{x + 1}$$

3. This one has a lot of missing terms, and so things will be slightly more challenging to keep organized:

$$\frac{x^5 - 1}{x + 1}$$

Solution:

$$\begin{array}{rr}
 & x^4 - x^3 + x^2 - x + 1 \\
 \cline{2-2}
x+1 \big) & x^5 \qquad\qquad\qquad\qquad\quad - 1 \\
 & -x^5 - x^4 \qquad\qquad\qquad\qquad \\
 \cline{2-2}
 & -x^4 \qquad\qquad\qquad\qquad \\
 & x^4 + x^3 \qquad\qquad\qquad \\
 \cline{2-2}
 & x^3 \qquad\qquad\qquad \\
 & -x^3 - x^2 \qquad\qquad \\
 \cline{2-2}
 & -x^2 \qquad\qquad \\
 & x^2 + x \qquad \\
 \cline{2-2}
 & x - 1 \\
 & -x - 1 \\
 \cline{2-2}
 & -2
\end{array}$$

So, we have

$$\frac{x^5 - 1}{x + 1} = x^4 - x^3 + x^2 - x + 1 - \frac{2}{x + 1}.$$

I skipped some steps in this last calculation. The idea is the same as with the previous examples and once you've done a couple of these, you'll see where you can skip ahead to the answer.

4. This problem is much like the last, but slightly more complicated since the denominator has three terms instead of 2:

$$\frac{x^5 - 1}{x^2 + x + 1}$$

Solution:

$$\begin{array}{rr}
 & x^3 - x^2 \qquad + 1 \\
 \cline{2-2}
x^2+x+1 \big) & x^5 \qquad\qquad\qquad\quad - 1 \\
 & -x^5 - x^4 - x^3 \qquad\qquad \\
 \cline{2-2}
 & -x^4 - x^3 \qquad\qquad \\
 & x^4 + x^3 + x^2 \qquad \\
 \cline{2-2}
 & x^2 \qquad - 1 \\
 & -x^2 - x - 1 \\
 \cline{2-2}
 & -x - 2
\end{array}$$

So, we have

$$\frac{x^5 - 1}{x^2 + x + 1} = x^3 - x^2 + 1 + \frac{-x - 2}{x^2 + x + 1} = x^3 - x^2 + 1 - \frac{x + 2}{x^2 + x + 1}$$

5. The coefficients will be slightly larger in this problem:

$$\frac{3x^3 - 2x^2 + 4x - 3}{x^2 + 3x + 3}$$

Solution:

$$\begin{array}{rrrrr}
 & & & & 3x - 11 \\ \cline{2-5}
x^2 + 3x + 3\big) & 3x^3 & -2x^2 & +4x & -3 \\
 & -3x^3 & -9x^2 & -9x & \\ \cline{2-4}
 & & -11x^2 & -5x & -3 \\
 & & 11x^2 & +33x & +33 \\ \cline{3-5}
 & & & & 28x + 30
\end{array}$$

And so we have

$$\frac{3x^3 - 2x^2 + 4x - 3}{x^2 + 3x + 3} = 3x - 11 + \frac{28x + 30}{x^2 + 3x + 3}$$